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Composite plates with two concentric layups under compression

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Abstract

A new concept for designing composite panels with improved performance under compression is presented. In this concept, the panel consists of two different concentric layups. A Rayleigh–Ritz-based approach to model such rectangular panels under compression is presented. The buckling load and the in-plane stresses everywhere in the plate are determined using an energy minimization approach. The results are compared to detailed finite element models and, for special cases, to other published finite element solutions and are shown to be in good to excellent agreement except for cases where twisting–bending coupling (not accounted for by the present method) is significant. As shown by specific examples, the present method can be used to obtain lighter configurations than single-layup geometries. In addition, the method can be applied to plates with rectangular cutouts and plates with terminated stiffeners.

Keywords: A. Laminates; B. Buckling; B. Stress concentrations; C. Analytical modelling

1. Introduction

The buckling problem of composite plates under compression has been solved by various authors for different layup configurations and boundary conditions (e.g. [1–5]). With reliable analysis methods in place, the optimization problem where the optimum layup of a plate would be determined under various loadings was addressed using genetic algorithms [6–8], hit and run algorithms [9], neural networks with fuzzy logic [10] or combinations of other methods [11].

These methods help determine the optimum layup for lowest weight subject to various constraints. Still, the need for further reductions in weight calls for additional design refinements. One approach, Fig. 1, is to use more than one layups in the plate, for example having a different layup at the center of the plate than at the perimeter. A special case of this situation was examined by Biggers and Srinivasan [12] who showed that if the center layup extends to the edge of the plate, then judicious choice of the two layups and the edge dimension over which the perimeter layup extends (Fig. 1) leads to more than 200% improvements in buckling load for simply-supported plates.

Recently, Papadopoulos and Kassapoglou have examined the case of buckling of rectangular composite plates with two concentric layups under shear [13] and then generalized to any number of concentric layups [14]. They showed that weight savings as high as 26% are possible with judicious choice of the concentric layups and their respective sizes. An analogous approach as in [13,14] is applied here to plates under compression.

A method to predict the buckling load and stress distribution before buckling in rectangular plates with two concentric layups under compression is developed. The method can be used for design and analysis of such plates using only the applied load at the outer perimeter. This eliminates the need for modeling each of the two layups separately where the loads and displacements at their interface are used as boundary conditions for the inner layup. The method can also be used in optimization algorithms where the optimum combination of center and perimeter layups can be determined on the basis of the overall plate

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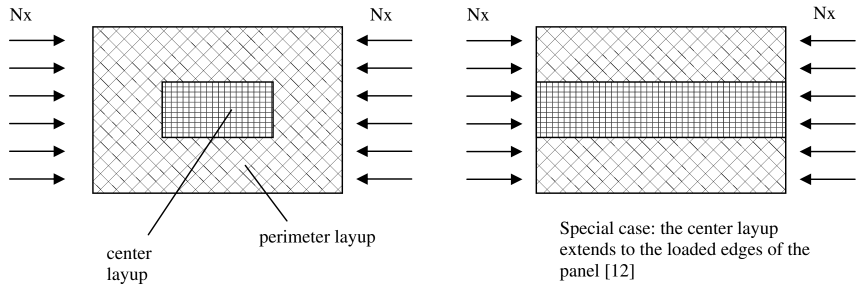

Fig. 1. Rectangular plate with two layups under compression.

dimensions and applied loading. Finally, it will be shown that, given a baseline single-layup configuration under compression, a combination of two layups can be created that meets the same applied load at a reduced weight.

2. Solution

2.1. Buckling of plates with two concentric layups under compression

Consider a rectangular plate with two concentric layups under compression (Fig. 2). The plate is assumed to be simply-supported at its edges.

The out-of-plane displacement w of the plate is assumed to be of the form

$$w = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} \sin m\pi\xi \sin n\pi\phi \tag{1}$$

where m and n are odd, and A_{mn} are unknown coefficients.

The normalized variables ξ and ϕ are given by

$$\xi = \frac{x}{2a}$$
$$\phi = \frac{y}{2b}$$

Also, for future use, define $\alpha = A/a$ and $\beta = B/b$. Assuming that both layups are symmetric and balanced with $D_{16} = D_{26} = 0$, the buckling load can be determined by minimizing the total potential energy Π of the plate:

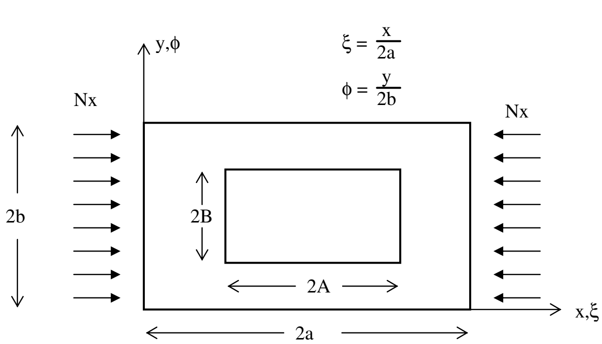

Fig. 2. Composite plate with two layups – problem geometry.

$$\begin{aligned}\Pi = \frac{1}{2}\int\int\Bigg\{ & D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2D_{12}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) \\ & + D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)\Bigg\} \mathrm{d}x\,\mathrm{d}y \\ & - \frac{1}{2}\int\int N_{\mathrm{a}}\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x\,\mathrm{d}y\end{aligned} \tag{2}$$

where N_{a} is the applied compressive load per unit width, and D_{ij} ($i = 1, 2, 6$), are the bending stiffnesses at any location in the plate. The integrations in Eq. (2) are carried out over the entire plate. Note that the coupling terms D_{16} and D_{26} could be incorporated with appropriate changes in Eq. (1) at considerable increase in complexity.

It should be noted that since the perimeter and center layup have different bending stiffnesses, the first term in Eq. (2) must be broken up in individual pieces each of which has a different value for the corresponding D_{ij} term. The integral limits in the second term in Eq. (2) are $(0, 2a)$ and $(0, 2b)$ or, using the normalized variables ξ,ϕ: $(0, 1)$ for both integrations.

Due to the fact that w in Eq. (1) is in terms of a product of a function of ξ multiplied by a function of ϕ, the integrations required for the first term in Eq. (2) can be written in a relatively simple manner. Each term has the form

$$T = \int\int D_{ij} f(\xi) g(\phi) \mathrm{d}\xi\, \mathrm{d}\phi$$

By denoting the bending stiffnesses for the center layup by $\overline{D}_{ij}$ and the bending stiffnesses of the perimeter layup by D_{ij}, and letting $F(\xi)$ and $G(\phi)$ be the integrals of f and g respectively, the integral over the entire plate can be written in the form

$$\begin{aligned}T = & D_{ij}(F(1) - F(0))(G(1) - G(0)) + (D_{ij} - D_{ij})\left(F\left(\frac{1+\alpha}{2}\right)\right. \\ & \left. - F\left(\frac{1-\alpha}{2}\right)\right)\left(G\left(\frac{1+\beta}{2}\right) - G\left(\frac{1-\beta}{2}\right)\right)\end{aligned} \tag{3}$$

Using Eqs. (3) and (1) to substitute in Eq. (2), the energy Π can be evaluated. Then, the unknowns A_{mn} are determined by minimizing the energy

$$\frac{\partial \Pi}{\partial A_{mn}} = 0 \tag{4}$$

Carrying out the integrations, and setting the derivative of Π with respect to A_{mn} equal to zero, leads to the generalized eigenvalue problem:

$$[\mathbf{A}]\{\mathbf{x}\} = \lambda[\mathbf{B}]\{\mathbf{x}\} \tag{5}$$

where the eigenvalue λ is the buckling load N_a in Eq. (2), the eigenvector {x} contains the constants A_{mn} of Eq. (1), and the matrices [**A**] and [**B**] contain stiffness and geometry information of the problem.

For the solution of the eigenvalue problem, standard algorithms [15,16] were used. It should be noted that if the stiffness of the center layup is drastically different than the stiffness in the perimeter, a relatively large number of terms in w in Eq. (1) is necessary for convergence. This corresponds to cases where the stiffer layup is almost rigid and a large number of terms in the series expression in w is needed to accurately model the transition from the softer part to the relatively rigid (relatively flat) portion of the plate. Convergence studies showed that 15–20 terms in x and y (total of 225–400 terms) were sufficient in the vast majority of cases.

2.2. In-plane stresses in plates with two concentric layups under compression

In addition to the buckling case, the membrane problem where the in-plane stresses prior to buckling may reach locally the failure stress of the material must also be solved. The stiffness discontinuity at the boundary between the center and perimeter layups causes local stress concentrations that can be significant.

The manner in which the load is introduced into the plate can be very important. For simplicity, only the case of constant applied force F_a is analyzed here. An analogous approach (with appropriate changes in Eq. (6) below) can be used for the case of uniform applied displacement.

The in-plane forces per unit length N_x, N_y, and N_{xy} are selected as follows:

$$N_x = \frac{F_a}{2b} + \sum_{m=1}^{M}\sum_{n=1}^{N} H_{mn}(\cos 2m\pi\xi - 1)\cos 2n\pi\phi$$

$$N_y = \frac{b^2}{a^2}\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{m^2}{n^2}H_{mn}\cos 2m\pi\xi(\cos 2n\pi\phi - 1) \tag{6}$$

$$N_{xy} = \frac{b}{a}\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{m}{n}H_{mn}\sin 2m\pi\xi\sin 2n\pi\phi$$

It can be shown that the expressions in Eq. (6) satisfy the force equilibrium equations

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0$$

and the force boundary conditions

$$N_x(x=0) = N_x(x=2a) = \frac{F_a}{2b}$$

$$N_y(y=0) = N_y(y=2b) = 0$$

$$N_{xy}(x=0) = N_{xy}(x=2a) = N_{xy}(y=0) = N_{xy}(y=2b) = 0 \tag{7a–c}$$

Eq. (7a) is an expression of the fact that a constant force F_a is applied at two opposite panel edges. Eqs. (7b) and (7c) simply enforce the stress-free conditions at the panel edges. It should be noted that both even and odd terms are used in the series expressions in Eq. (6), as opposed to the w expression Eq. (1) where only odd terms were used.

The unknown coefficients H_{mn} in Eq. (6) are determined by minimizing the energy in the panel

$$\Pi = \frac{1}{2}\int\int[a_{11}N_x^2 + a_{22}N_y^2 + 2a_{12}N_xN_y + a_{66}N_{xy}^2]\mathrm{d}x\,\mathrm{d}y - \int N_x(x=2a)u(x=2a)\mathrm{d}y \tag{8}$$

where a_{ij} are the corresponding entries of the inverse of the membrane stiffness matrix A_{ij} for the panel. It is assumed here that both center and perimeter layups are symmetric and balanced with $A_{16} = A_{26} = 0$.

In order to evaluate Eq. (8), the displacement u at the panel edge ($x = 2a$) must be determined. This is done by calculating the strain ε_x from

$$\varepsilon_x = \frac{N_xA_{22} - N_yA_{12}}{A_{11}A_{22} - A_{12}^2} \tag{9}$$

with A_{ij} the corresponding entries of the membrane stiffness matrix and N_x, N_y given by Eq. (6).

Integrating Eq. (9) with respect to x gives the displacement u to within a function of y. This function of y is determined by the symmetry condition $u(x=0) = 0$. The final expression for $u(x=2a)$ is

$$u(x=2a) = u(\xi=1) = \frac{2a}{A_{11}A_{22} - A_{12}^2}\left(\frac{A_{22}F_a}{2b} - A_{22}\sum_m\sum_n H_{mn}\cos 2n\pi\phi\right) \tag{10}$$

The unknown coefficients H_{mn} in Eqs. (6) are obtained by energy minimization

$$\frac{\partial \Pi}{\partial H_{mn}} = 0 \tag{11}$$

Using Eqs. (10) and (6) to substitute in (8) and then in (11), gives the system of equations

$$[\mathbf{E}]\{\mathbf{H}\} = \{\mathbf{R}\} \tag{12}$$

where the vector {H} contains the unknowns H_{mn}, the matrix [**E**] is obtained from carrying out the integrations of the first term in Eq. (8) along with the corresponding differentiations of Eq. (11), and the right hand side vector {R} is obtained from the second term in Eq. (8).

The system of Eq. (12) can be solved by any Gaussian elimination algorithm.

As was mentioned in presenting the solution to the buckling problem, the stiffness discontinuity at the interface between the two layups requires a relatively large number of terms to obtain a converged solution in particular at the four corners of the center layup. Thirty terms in x and thirty in y (giving a system of 900 equations in 900 unknowns in Eq. (12)) were shown to work well in most cases.

3. Comparisons to finite element results and other solutions

The approach developed in the previous sections was compared to finite element predictions. For the buckling analysis, a 50.8 cm × 50.8 cm square panel was selected. The center layup was $(\pm 45)_5/(0/90)_2/(\pm 45)_5$ and the perimeter layup was $(\pm 45)/(0/90)_2/(\pm 45)$. The material properties were typical of plain weave fabric with $E_{11} = E_{22} = 67.5$ GPa, $G_{12} = 4.48$ GPa and $\nu_{12} = 0.05$. Six different cases were run, each with different dimensions for the center layup: (a) 5.08 cm × 5.08 cm, (b) 10.16 cm × 10.16 cm, (c) 25.4 cm × 25.4 cm, (d) 38.1 cm × 38.1 cm, (e) 45.72 cm × 45.72 cm, and (f) 50.8 cm × 50.8 cm. In the last case, the entire panel was made of the center layup and was used as a check against exact solutions [e.g. 17].

For the finite element model, the NEiNastran software by Noran Engineering was used. The model consisted of 14,400 four-noded quadrilateral elements with 14,641 nodes. The geometry, loading, and typical results for case (c), 25.4 cm × 25.4 cm center layup, as obtained from the finite element model are shown in Fig. 3.

As is seen from Fig. 3, the buckling mode predicted by the finite element model is not symmetric. The highest

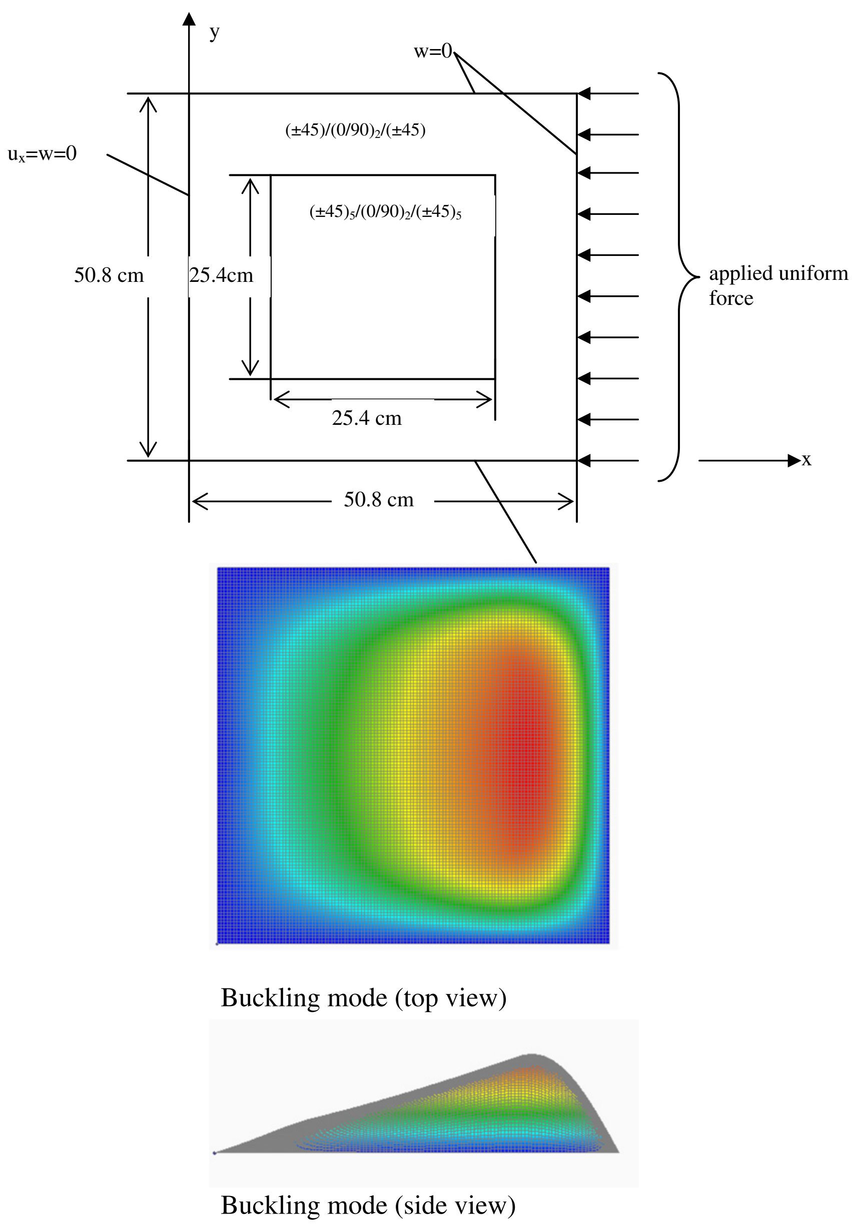

Fig. 3. FE-predicted buckling mode for case (c): 25.4 × 25.4 cm center layup.

out-of-plane deflections do not occur at the panel center but nearer the loaded edge ($x = 50.8$ cm). This is due to the fact that a constant force was applied at $x = 50.8$ cm instead of a constant displacement. The mismatch in stiffness between the center and the perimeter of the panel results in non-uniform strains across the panel at any x value. A straight line at $x =$ constant before loading becomes curved with higher displacements near the panel edges ($y = 0$ and $y = 50.8$) and lower displacements at the panel center when the center of the panel is stiffer than the perimeter. The $x = 0$ edge of the panel is not allowed to move in the x direction while the $x = 50.8$ cm edge moves following the application of the constant force as described. Because the edge $x = 0$ is stationary, the portion of the panel near that edge is more rigid and deforms less than the portion of the panel near the loaded edge which is allowed to move freely in the x direction. As a result, when the panel buckles, the softer part of the panel (near the loaded edge) will deflect more. Another way to see this is to consider the exact same problem but instead of one edge loaded and the other stationary, now both edges are loaded in compression. This case is completely symmetric and the buckling mode is completely symmetric as predicted by the finite element solution and shown in Fig. 4. The buckling loads for the cases in Figs. 3 and 4 differ by 6.5%. Finally, if a constant displacement were applied instead of a constant force, the buckling mode would still be symmetric as was verified by finite elements.

The buckling loads predicted by the method in Section 1 are compared in Table 1 and Fig. 5 to the predictions

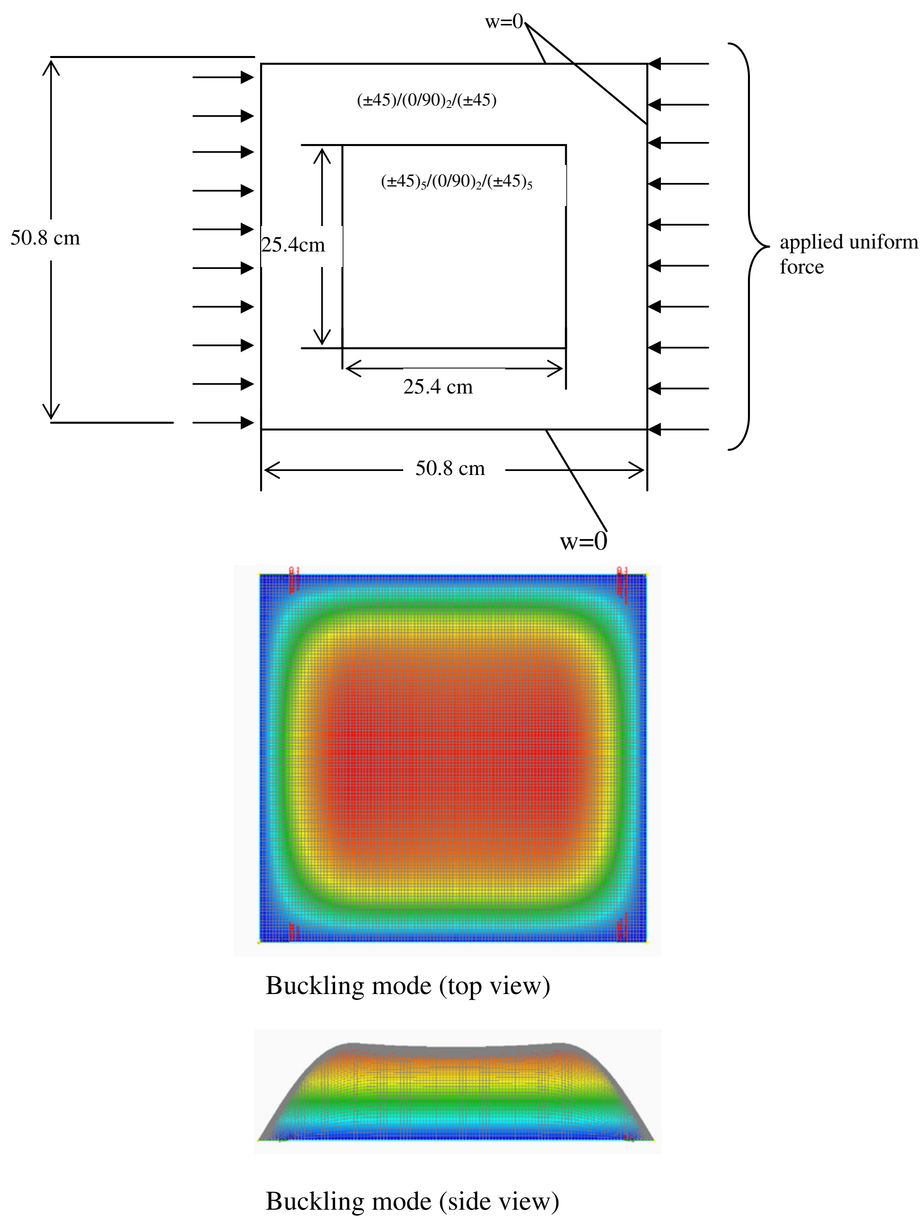

Fig. 4. FE-predicted buckling mode for case (c) when both panel edges are under constant compressive load.

Table 1
Buckling load predictions compared to finite element results

Center layup dimensions (cm)	Prediction by present method (N/m)	Finite element prediction (N/m)	% Difference
5.08 × 5.08	379.8	397.5	−4.5
10.16 × 10.16	424.6	428.3	−0.01
25.4 × 25.4	693.3	682.0	+1.7
38.1 × 38.1	1748.1	1636.3	+6.8
45.72 × 45.72	6459.2	5686.1	+13.6
50.8 × 50.8	10,288	10,157[a]	+1.3

[a] Exact buckling load using [17] = 10,253 N/m.

obtained from the finite element model. In all cases, the FE-predicted buckling mode was unsymmetric. This asymmetry was less pronounced when the center layup was either a very small or a very large portion of the entire panel. The highest difference between the two methods is less than 14% with all other cases within 7%. Using more than 20 terms in x and y in Eq. (1) would improve the agreement further. Considering the fact that the mismatch in bending stiffness between the center and perimeter layups is more that an order of magnitude (center stiffness ≈ 25 times perimeter stiffness) the agreement is considered very good in most cases.

The same finite element model was used to predict the in-plane loads N_x for a constant applied force and compare them to the results of Section 2. This was done for cases (a), (b), and (c). The results are shown in Fig. 6. Excellent

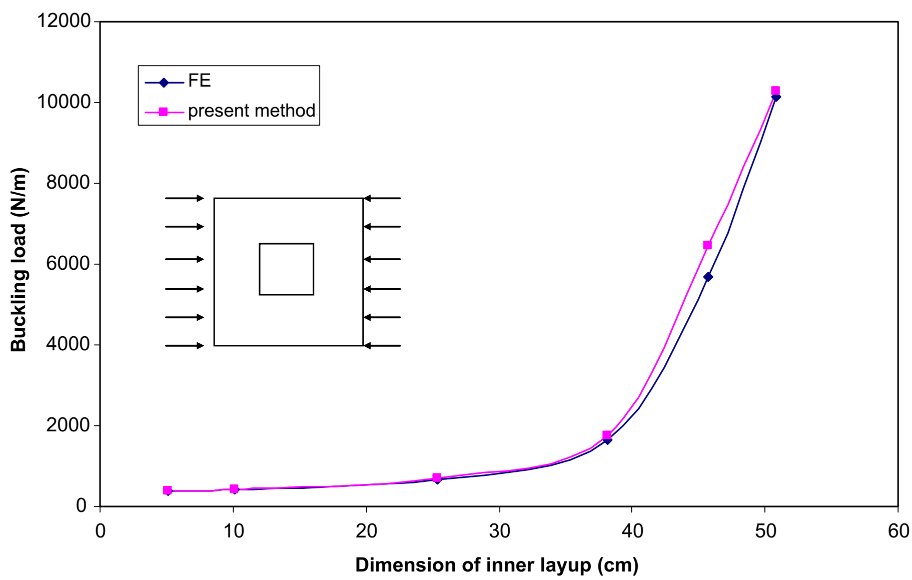

Fig. 5. Buckling load for inner layups with different sizes – present method versus finite element results.

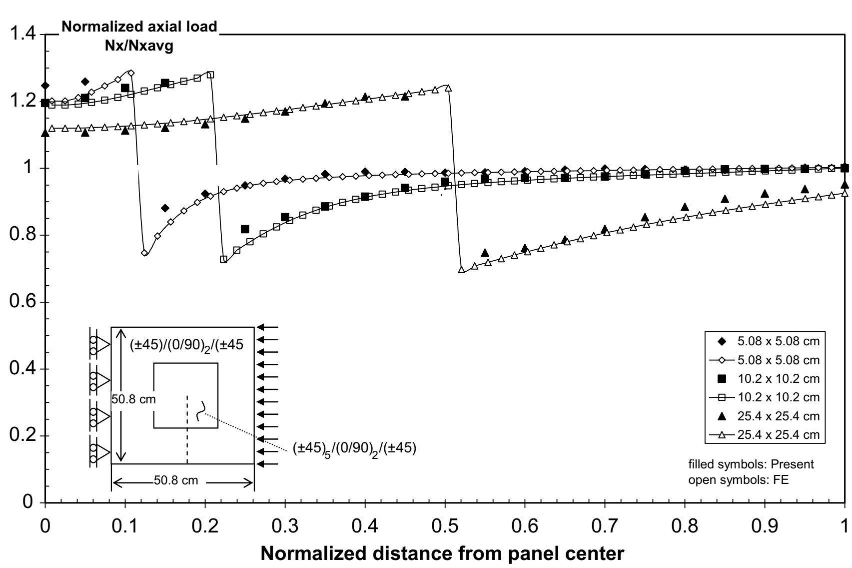

Fig. 6. Axial load N_x at panel center – present method versus finite element results.

agreement between the present method and the finite element results is observed. The present method can be used to obtain the stresses anywhere in a plate with concentric layups, in particular at the interface between layups where the stress concentrations can be significant. Accounting for these stress concentrations in design is necessary to avoid premature failures.

Finally, a comparison to the finite element predictions for the buckling load by Biggers and Srinivasan [12] for the special case of Fig. 1 is presented. The present results are compared to those in Fig. 4 of [12] for the simply-supported case. It should be noted that this is a case where the layups used ([±45/0$_n$/90]s at the edges and [±45/90]s at the center) have nonzero bending–twisting coupling terms D_{16} and D_{26} terms. In addition, transverse shear effects are included in [12] but not in the present analysis. Therefore, the present method is expected to depart from the results in [12] where these effects are significant. The comparison of the two methods is shown in Fig. 7. The buckling ratio is the ratio of the buckling load for each case in question to a baseline buckling load where the entire panel is taken up by a single layup, [±45/0/90]s. The width ratio b_1/b is allowed to vary continuously in order to obtain the different comparison cases exactly as was done in [12].

For width ratios between 0.1 and 0.25, the present method departs significantly from the results in [12]. The main reason is that the [±45/0$_n$/90]s layup is confined to a narrow strip at the edges and the majority of the center is taken up by [±45/90]s for which the twisting–bending coupling terms D_{16} and D_{26} are significant. As already mentioned, the present method does not account for twisting–bending coupling nor for transverse shear effects. For width ratios greater than 0.3, the present method is in excellent agreement with the results in [12]. Here, the central portion is narrower and the twisting–bending coupling effects are much smaller so the error in the present method is negligible.

4. Applications

Besides the obvious application already desribed by Biggers and Srinivasan in [12], with the associated buckling load improvements shown in Fig. 7, the present method can also be used to improve the buckling loads by using a "patch" like layup in the middle of a plate. Some results showing potential improvements are shown in this section.

Consider a square plate with edge 50.8 cm. Three cases are examined: In the first, the baseline layup, covering the entire plate, is $(\pm45)/(0/90)_2/(\pm45)$. In the second, it is $(\pm45)/(0/90)_3/(\pm45)$ and in the third, $(\pm45)/(0/90)_4/(\pm45)$. The material is plain weave fabric with the properties given at the beginning of the previous section.

For each of the cases, a configuration with two concentric square layups is sought that has the same weight as the baseline but higher buckling load. With reference to Fig. 2, the dimension of the center layup that gives the same panel weight as the baseline can be shown to be (assuming all plies have the same thickness)

$$A = a\sqrt{\frac{n_{\mathrm{base}} - n_{\mathrm{perim}}}{n_{\mathrm{ctr}} - n_{\mathrm{perim}}}} \qquad (13)$$

where n denotes number of plies and the subscripts refer to the baseline, perimeter and center respectively.

For simplicity, the perimeter layup is taken to be $(\pm45)/(0/90)/(\pm45)$ for all cases. The center layup is $(\pm45)/(0/90)_n/(\pm45)$ with n appropriately chosen. For each selection of n, the dimension of the center layup can be determined so the two-layup configuration has the same weight as the baseline. Then, the method described in section (1) for determining the buckling load is used to compare the buckling load of the two-layup configuration to the baseline. This is repeated for various values of n. Typical results are shown in Table 2.

As the baseline becomes thicker, the % improvement increases. At the same time the amount covered by the cen-

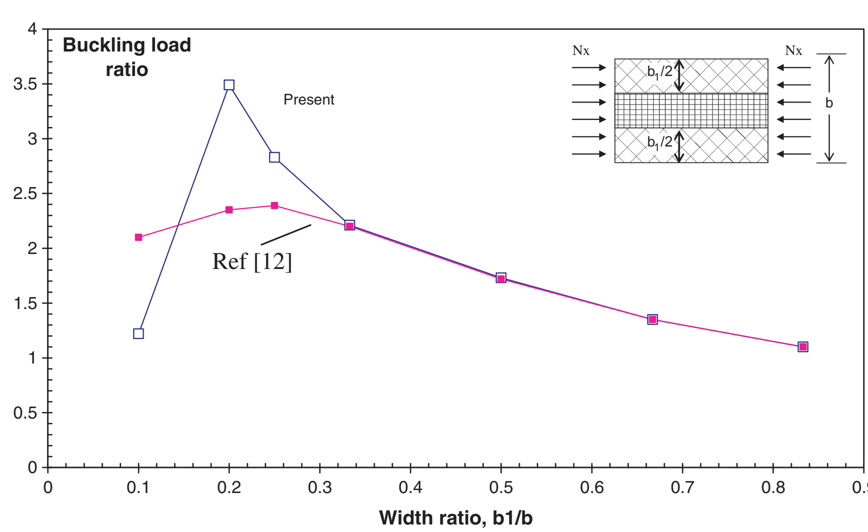

Fig. 7. Special case where perimeter layup extends to panel edges: comparison with [12].

Table 2
Buckling load improvements using two concentric layups

Baseline layup	Buckling load of baseline (N/m)	Center layup	Dimension of ctr layup to match baseline wt. (cm)	Buckling load of two-layup configuration (N/m)	% Increase
$(\pm45)/(0/90)_2/(\pm45)$	362.2	$(\pm45)/(0/90)_3/(\pm45)$	35.92	372.6	2.9
$(\pm45)/(0/90)_3/(\pm45)$	680.0	$(\pm45)/(0/90)_4/(\pm45)$	41.47	736.0	8.2
$(\pm45)/(0/90)_4/(\pm45)$	1132.4	$(\pm45)/(0/90)_5/(\pm45)$	43.99	1280.6	13.1

Perimeter layup: $(\pm45)/(0/90)/(\pm45)$.
Panel dimensions: 50.8 cm × 50.8 cm.

ter layup increases. For example, in the last case in Table 2, the perimeter layup covers only 3.4 cm all around the perimeter raising some questions about how practical such a design would be. In fact, as was found for the applied shear case in [14], it is expected that there will be a range of thicknesses and areas of the center layup beyond which the improvements in buckling load will only be marginal. Still, the improvement in the buckling load can become significant. A more rigorous optimization approach or detailed study as the one in [14] would give a better idea of the potential improvements using this concept. In addition, blending of the layups using an approach similar to the one in [8] will assure load continuity across layups and will help minimize stress concentrations.

Some other special cases to which the method can be applied are of great significance because of their frequency of occurrence in practice. These are shown in Fig. 8 and are as follows:

1. The case where the center layup extends to the edges of the panel. Besides the case of a plate with two reinforcing strips at its two unloaded edges, this also covers the case of a plate with two stiffeners at the edges. As long as the stiffeners are represented by equivalent bending stiffnesses D_{11}, D_{12}, D_{22}, and D_{66}, the method is directly applicable.
2. A case mirroring case 1 above is that of a single stiffener in the middle of the plate. The stiffener becomes the center layup again with appropriate bending stiffness values.
3. Terminated stiffener (e.g. 18). The present method has the additional advantage of providing a solution to the case of a terminated stiffener in the middle of the plate. If the stiffener does not extend to the loaded edges of the plate, it is analogous to a center layup with stiffness values representing the terminated stiffener. Here, the in-plane stress solution can be useful since it will give an indication of the stress concentration at the stiffener termination.
4. Plate with rectangular cutout. By setting the appropriate stiffness values of the center layup to zero, the present method will provide solutions for composite plates with rectangular cutouts. Detailed solutions for such cases have been developed in [19–21]. The present method provides an efficient way to address this problem.
5. Panel breaker condition. Often, stiffeners are used as panel breakers where their role is to break plates into smaller sections thereby increasing their buckling loads. There are several design guidelines [22,23] for coming up with the stiffness requirements for such stiffeners. The present method allows detailed determination of the buckling load of a plate with a central stiffener. The

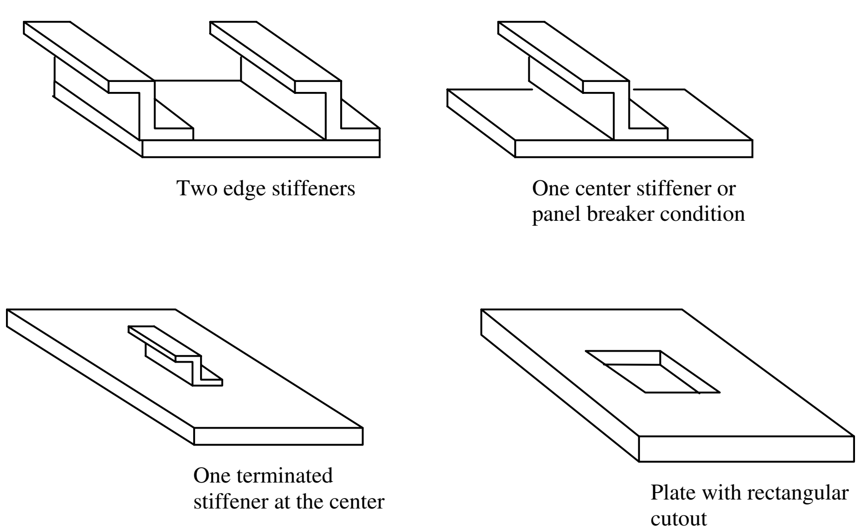

Fig. 8. Special cases approximated by the present method.

stiffness of the central stiffener can then be changed until the buckling load of the plate with the stiffener equals the buckling load of each of the individual smaller plates created when the stiffener is placed in the middle. In that case, the stiffener acts as a panel breaker.

5. Summary and conclusions

The method presented allows the determination of the buckling load and in-plane stresses in composite plates with two concentric rectangular layups. The method is based on a Rayleigh–Ritz formulation and leads to a generalized eigenvalue problem for the buckling case and a linear system of equations for the in-plane stresses. The method was shown to be in very good agreement with finite element solutions developed here or in the literature, with the exception of cases where transverse shear and twisting–bending coupling effects are significant.

The method can be used to analyze stiffened panels with one stiffener in the middle or two stiffeners at the edges, stiffened panels with one terminated stiffener at the center, or plates with rectangular cutouts. It can also be used to determine the stiffness requirements for stiffeners that are to be used as panel breakers.

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